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AN ANALYSIS OF ETHNOMATHEMATICS DIDACTICAL CONTRACT IN THE SUMS OF INTERIOR AND EXTERIOR ANGLES

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Abstract

Teachers and students have different ideas and expectations about the didactical situation that evolves in the classroom. It needs rules that determine teachers' and students' unique responsibilities relating to the didactical situation or contract. The division of roles and responsibilities moves from the teacher as the centre to the students as the centre. The results of a case study design of 42 students showed that the transition from ostentation to Mayeutic Socrate and then to the adidactical contracts was smooth and progressive if the teacher and the participants cooperated and actively participated in the milieu. It was therefore recommended that teachers streamline method-friendly guidelines, strategies, and techniques at the ostentation contract for students to emulate, students cooperate and support the teacher at the mayeutic Socratic contract and the knowledge of both local indigenous and exotic artifacts must complement each other at the adidactical contract.

Keywords: Didactical contract, Exterior angles, Interior angles, Students, Sums.

Abstrak

Guru dan siswa memiliki gagasan dan harapan berbeda tentang situasi didaktis yang berkembang di kelas. Hal ini memerlukan peraturan yang menentukan tanggung jawab unik guru dan siswa terkait dengan situasi atau kontrak didaktis itu. Pembagian peran dan tanggung jawab berpindah dari guru sebagai pusat ke siswa sebagai pusat. Hasil dari desain studi kasus dengan 42 siswa menunjukkan bahwa transisi dari ostentasi ke Mayeutic Socrate dan kemudian ke kontrak adidaktik berjalan lancar dan progresif jika guru dan peserta bekerja sama dan berpartisipasi aktif dalam lingkungan tersebut. Oleh karena itu, disarankan agar guru menyederhanakan pedoman, strategi, dan teknik yang ramah metode pada kontrak ostentasi agar dapat ditiru oleh siswa, siswa bekerja sama dan mendukung guru pada kontrak Mayeutic Socrate, dan pengetahuan tentang artefak lokal dan asing harus saling melengkapi pada kontrak adidaktik.

Kata kunci: *Jumlah, Kontrak didaktik, Siswa, Sudut luar, Sudut dalam.*

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INTRODUCTION

The sum of interior and exterior angles has been a matter of concern in the mathematics curriculum. The Ministry of Education (2019) has brought in new innovative pedagogies to help improve students in this area. However, the concern is whether the in-service teachers have welcomed one of the novel approaches and resources. The urgency required that learners integrate ethnomathematics pedagogies to support what they require to

achieve their goals.

The complement of teachers' mathematical training is reflected in the updating of in-service teachers and the programmes and preparation of new teachers in teacher training Colleges of Education. Also, important is its activity in the production of support materials for the work of teachers in the classroom, i.e. in the creation of mathematics texts, worksheets for students, didactic games and toys, collections of problems and exercises, and less on sequences (Hurtado et al., 2021). One of the Theories of Didactical Situation (TDS) frameworks is to integrate the epistemology of these materials.

To integrate the epistemology of the materials, we require three types of situations: action, formulation, and validation. The action situation helps students to determine the strategy, the formulation situation helps students to interact in class, and the validation situation equips students to develop, revise, and infer mathematics arguments. Supporting cognitive hypotheses related to the materials are the cognitive dimensions of the processes of adaptation and acculturation. Adaptation helps students to learn by adjusting themselves to their environment in the wake of contradictions, differences, and imbalances in the classroom milieu. Acculturation helps students to connect the construction of their knowledge with other students in the class. We require the three types of situations and the two cognitive dimensions of the hypothesis to form the genesis of the didactical contract (Maknun et al., 2020).

The didactical contract

In a teaching process, teachers, and students each have different ideas and expectations relating to the didactical situation that develops in the classroom. The classroom requires rules to determine a teacher and student’s responsibility. In real-life situations, the relevance of division and assigning of duties and responsibilities is non-negotiable. The division of roles and responsibilities in the classroom interaction is referred to as a didactical contract. It can span from the teacher as the centre to the students as the centre, and vice versa (Prabawanto et al., 2018). This didactical contract ensures that teachers progressively scaffold learners in the mathematics classroom. As innovations are necessary, learners must also be able to devise and deploy any tool be it cultural or exotic, to the realisation of their set goals.

Three types of didactical contracts have been proposed. These are ostentation, mayeutic Socratic, and potential adidactical. The ostentation contracts represent the

teaching process, where the teacher is the main actor in explaining the concept, demonstrating the application of the concept, providing examples, as well as preparing the exercises. The mayeutic Socratic contracts represent a teaching process where the teacher does not completely dominate but helps students through the submission of the key questions following didactic situations that develop. A potential adidactical contract is a didactic contract, where the teacher provides space for the student as much as possible through their capacity, to encourage critical and independent thinking through the teacher's didactical design. In practice, these three types of didactical contracts occur alternately as needed in the design model (Prabawanto et al., 2018).

In addition, in the implementation process, termination of the didactical contract can occur (contract breaking) when there is a mismatch between the expectations of the teacher and the response shown by students during the class instruction. For teachers who want to continue, the situation of termination of the contract may be a trigger for improvements in design and appropriate didactical intervention model (Prabawanto et al., 2018).

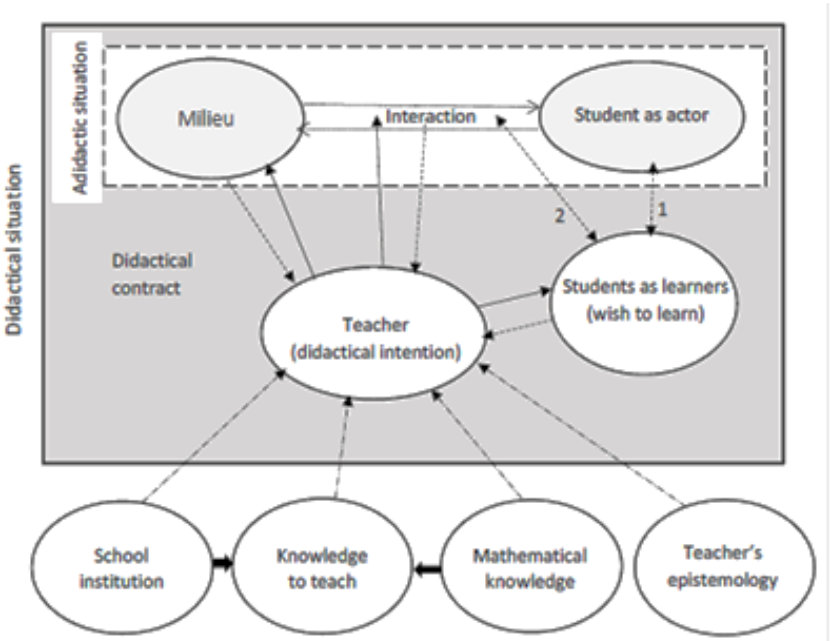

Figure 1. Didactical contract
(adapted from Mangiante-Orsola et al., 2018)

In Figure 1, students first build their knowledge through experience and interaction with their milieu in the situations of action, formulation, and validation. These three situations as tied in the interaction between teachers and students are known as the didactical contract (Fuadiah, 2021, 2017).

The didactical situation is below the milieu. The teacher has an intention to teach some mathematical objects and the students, which are linked by the didactical contract. The white rectangle inside the grey one (with a dotted edge) represents the adidactical situation. Inside the didactical situation is a way to learn a new piece of mathematical knowledge: a generic student, representing any student, acts in a milieu that can give feedback on those actions. The adidactical situation may be considered as a game defined by this milieu, rules to interact with it and an aim to reach (Nery, 2022). It is constructed or chosen by the teacher such that the knowledge to gain will be the knowledge to be learned and the prior knowledge of students may help them to learn and interpret the results of the milieu (Mangiante-Orsola et al., 2018).

The conditions can be expressed by three constraints on the milieu (Mangiante-Orsola et al., 2018): (1) to provoke contradictions, and difficulties for the students so that they have to adapt their knowledge; (2) to allow them to work autonomously; (3) to help them to learn some specific mathematical content (by learning to learn the proof).

Thus, to learn, the student has to learn the proofs (acting him/herself or in interaction with others), follow the rules (and his/her idea), and reflect on this action taking into account the feedback of the milieu, whether s/he understood or stacked.

Along the black arrows, the teacher interacts with the milieu (to construct it before the class or to modify it during the class), eventually with the relation between the actor and the milieu to change the knowledge (with an aim of devolution for instance) or on the students' knowledge (institutionalization for instance). Along the dotted arrows, the teacher takes information on the relationship between the student and the (adidactical) milieu, on the students' knowledge (acted or expressed). S/he will be able to use this information to modify the milieu or to give some help to some students. The students as learners consider the action in the milieu (arrows 1 and 2) and reflect on it as a way to produce new knowledge. These actions may be indirect or implicit (not easy to observe). The arrows with short lines and dots (at the bottom of Figure 1) represent the constraints and objectives of the teacher, coming from the school institution or her/himself. Knowledge to teach is interpreted by the teacher from the curriculum and her/his mathematical knowledge. We do not represent constraints on students, though they exist, coming for instance from their parents or other students.

Didactics of triangle's sums of interior and exterior angles

Didactics of Mathematics provides a set of categories and subcategories of knowledge that the teacher must know how to apply and value (Prabowo et al., 2021). Teacher's didactic-mathematical knowledge is underpinned by common content knowledge, extended content knowledge, and specialized knowledge. Common knowledge refers to the knowledge that the teacher must put into practice to solve problematic situations about a specific mathematical topic (theory of the triangle). Extended knowledge identifies possible generalizations of the tasks and connections to the students. Specialized knowledge is directly linked to the teacher profession (Blanco et al., 2022).

Specialized knowledge is extended to specialized content knowledge, content knowledge concerning students, content knowledge about teaching, and content knowledge with the curriculum. Specialized content knowledge involves the identification, by the teacher, of the content involved in a given problem situation. Content knowledge refers to the teacher's ability to develop strategies that help students solve problems, describe, and resolve students' learning conflicts, and generally, be able to describe the types of cognitive configurations that students present when solving a problem situation. Content knowledge in teaching implies reflection by teachers on the teaching and learning processes. Content knowledge with the curriculum requires that the teacher contemplates the adequacy of the content implemented with the curriculum, the timing, and everything that has to do with the context in which the teaching-learning process occurs (Blanco et al., 2022).

Research questions

The following research questions guided the study: (RQ1) How do students employ ostentation didactical contracts?; (RQ2) How do students use mayeutic Socratic didactical contracts?; (RQ3) What do students use in mayeutic potential adidactical contracts?

RESEARCH METHODS

The researcher used the case study design to structure and analyze the phenomenon. According to Priya (2021), case studies have become the most dominant designs for research because they involve data collection and not just methods, detailed analysis, allow for any method of data collection, and events or phenomena. There are three main types of case study designs, namely descriptive, explanatory, and exploratory. The

research explores the descriptive case study design to narrate step by step the phenomenon of finding the sums of interior and exterior as the design is best suited for anthropology and didactics (Priya, 2021).

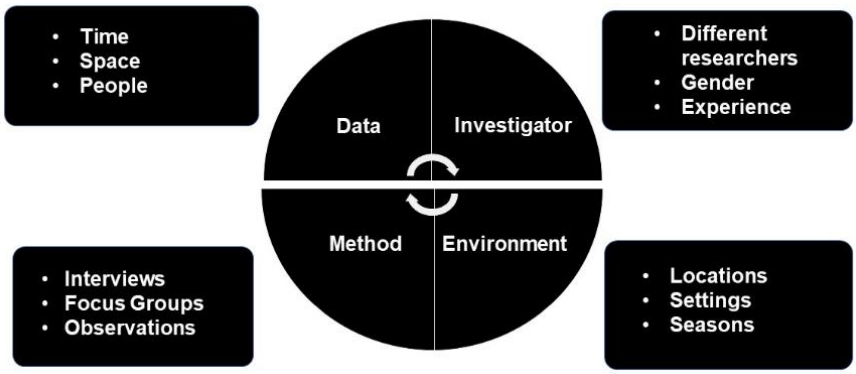

Figure 2. A case study descriptive design

In the design, the main concerns were the data (time, space, and people), method (interview, focus group, and observations), environment (locations, settings, and seasons), and investigator or researcher (gender and experiences). All these concerns have been addressed in this study.

Population and sample

In qualitative research, only a sample of a population is selected for any given study. However, the study's research objectives and the characteristics of the study population (such as size and diversity) determine which and how many people to select. Another critical issue is the selection or eligibility criteria, including inclusion, exclusion, and elimination criteria, which help delineate the eligible population (Creswell & Creswell, 2018). Even though sample size determination in determining qualitative sample size apriori is problematic, diversity and eligibility necessitated a full description and determination of the population of this study.

Willie (2023) refers to the entire group of individuals or elements that share a common characteristic or feature. It represents the larger, comprehensive group that is the focus of a study or analysis (Bhandari, 2023). For instance, because the study was on the sum of interior and exterior angles, the population encompassed all students in that Junior High School who were potentially affected by the teaching and learning of Polygons within the Effutu Municipal District. There were about 114 students. The target

population refers to a specific subset or segment within the larger population that is the primary focus of a study, intervention, or strategy (Willie, 2023). It represents a narrower group of individuals who possess specific characteristics or meet certain criteria (Bhandari, 2023). The identification of the target population was based on the research question for the problem. In this study, the target population was based on the number of students in Junior High School Form 3. This is the specific group for whom the Sum of Interior and Exterior Angles was intended to be taught at the grade level. There were 42 students.

A case study design is generally a qualitative work. All the 42 students were taken as the sample. Purposive techniques were used to select the participants. The purposive sampling intentionally selected the individuals who made interesting and thought-provoking responses to learn or understand the phenomenon. This enabled the researchers to handpick the cases to be included in the sample based on their judgment and typicality (Bhandari, 2023).

The sample comprised girls (22) and boys (20). The sample also cut across students from rural (12), peri-urban (15), and urban (15). In addition, students came from diverse social, economic, cultural, and religious backgrounds as they came from all settlements of the Effutu municipality and beyond. According to Jacobs (2018), the classification of qualitative samples into groups constitutes a necessary evil in the construction of an efficient policy aiming at equal opportunities for all. Jacobs (2018) cautions that case selection should be avoided on the dependent variable but be based on the independent (explanatory) factors provided we want to draw any inference and generalize the findings. In this way, the researcher built up a sample that is satisfactory to specific needs to ensure the representativeness, trustworthiness, and confirmability of the findings (Creswell & Creswell, 2018).

Data collection procedure

In all 42 students signed the consent forms to participate in the study. All those who did not sign the consent forms were excluded from the study. The data collection proceeded in three distinct phases according to the three types of situations, namely action, formulation, and validation. To ensure the trustworthiness and confirmability of the sample, no identifiers and personal names were written. All the scripts were numbered as P1, P2, P3, and so on.

A set of 10 items involving the Sum of Interior and Exterior Angles were given. The researcher led the students to understand each of the three types of didactical situations. After the processes, the students were allowed to emulate the steps spelts out by the researcher. Then a sample of two tasks each were drawn from the three types of situations for analysis and discussion.

In modern research, sustainability and impact are essential to ensure that the findings bear significance on theory, methodology, practice, and theory. Torres et al. (2024) contend that research sustainability and impact are necessary for research and policy discussions. The researcher addressed methodological consistency by carrying out the data analysis under the perspective of the interpretative paradigms., taking into consideration, theorems and the attendant rigorous proofs. This helped to discover the relationships between the findings and research sustainability in Sums of Exterior and Interior Angles (Torres et al., 2024).

The theory was also time-tested and viable. The theorem of didactical situations needed no verification and validation. Instead, the researcher applied just the three types of situations to test their positive tenets and practicability in Polygons (Torres et al., 2024). The qualitative analysis was detailed enough as it contained the organization, coding, categorization, inference, and production of excerpts (Torres et al., 2024).

Research impact was massively felt on the number of references. Many reference authors emanated from the Sums of Interior and Exterior Angles and the Didactical Theory. The transformation of theorems and proofs into the three types of situations in phases could boost everyday knowledge and practice by stakeholders along the research's lifespan. The cogent and plausible critiques in the academic value chain are enormous supports to interaction and engagement between the academic and non-academic communities to encourage more studies; and emphasize research utility, and research impact (Torres et al., 2024).

Finally, "real impacts" are related to long-term effects, changes, or benefits and not to those that only last for some time. Even though the findings from the students generated impacts, larger sample sizes, areas in mathematics, and the participants involved could take a bit longer to generate the same impacts (Torres et al., 2024). We therefore admonish potential replications to do diligence to achieve better successes. This study was therefore open to potential replications with diligence to achieve better success.

Data collection instruments

The main qualitative data collection instruments for this research were the focus group discussion, interviews, and observations. The instruments were structured in four sections based on the three types of didactical situations. The first section collected demographic and background characteristics of respondents, the second section collected on the ostentation contracts, the third section evaluated the impact of the mayeutic Socratic contracts, and the fourth section collected data on the potential adidactical contracts.

The test items were open-ended (or workout) items. These allowed for the collection of a large number of theorems, administering more items in a short period, and providing proofs quickly and reliably. However, they do not allow one to know how students arrived at the answer. Thus, the answer could be correct for the wrong reason (Principe, 2022).

Principe (2022) posits that qualitative data collection methods range from interviews, observations, focus groups, and document analysis. The researchers explored the document analysis to examine written and visual transcripts of the students. This helped to provide insights into their thoughts, feelings, and experiences in Sums of Polygons. The tools of qualitative data include digital recorders, transcription software, note-taking tools, online platforms, and visual analysis software. Taking cognizance of didactical situations, the researcher implored transcription and visual analysis tools. In the transcription software, the codes and themes were generated based on the theorems we collected and examined. These were inputted and generated by statistical software. Because some parts of Sums of Polygons demand the use of diagrams and pictures, we also added cultural artifacts to follow the patterns and relationships students generated.

There were two main categories of biases that likely affected our data collection instruments. These were respondent and researcher biases. Respondent bias happens when the participants are not accurate reflections of their thoughts and feelings. In other to reduce respondent biases, we did not align students' answers regarding sensitive or controversial questions in a socially acceptable way (social desirability bias). The researcher neither motivated the students nor gave them leading questions to enable them to answer. To avoid the respondents' biases, the researcher rather asked indirect questions, and open-ended questions, and gave away sponsor details. To avoid researcher biases, the researcher did not interpret the data in a manner that supported the research questions while removing any unfavourable respondents (Principe, 2022).

Potential Scalability and Adaptability of the didactical situation were gaged by the

three different pathways to scaling up namely expansion, replication, and spontaneous. Expansion took place in our intentional and planned scaling up of the items. Replication took place in the intentional and planned scaling up of the model by reproducing tested task sheets in addition to the theorems. Spontaneous took place in the unintentional process of expansion or replication of the model by progressing from one stage to the other (Vaughan-Lee et al., 2018).

Data analysis procedures

The qualitative phase was analyzed by transcribing the interviews and coding key data elements. Then, the codes merged to form plausible thematic areas in tandem with the concepts in the Sums of Interior and Exterior Angles. Supported by online portals and blogs, excerpts of the transcripts were scanned and pasted in appropriate places for comparative analysis (Creswell & Creswell, 2018).

Reliability and validity

Trustworthiness of research is important if its findings are to be accepted and relied on. Reliability is therefore the deciding factor for determining the quality of a research. Chetty and Thakur (2020) contend that reliability tests for qualitative research can be established by techniques of refutational analysis, the use of comprehensive data, constant testing and comparison of data, the use of tables to record data, as well as the use of inclusive deviant cases. These techniques helped the researcher to support the data sourcing, data validation, and data presentation process of the research, as well as support the claim of reliability in terms of form and context. Triangulation also became very important in establishing reliability in the research. The use of the simple quantitative transcripts completely created a very positive attitude towards the overall concept of the research and helped to establish reliability in a much easier form.

The validity of this research was established using data blinding and the inclusion of different sampling groups in the plan. The research included artifacts of Adinkra, Egyptian pyramids, and Larabanga mosque in three phases to reduce biases. The sample included various social and religious groups, who have been mostly raised in rural and peri-urban environments, along with young, middle-aged, and elderly parents or guardians who have had a partial upbringing in the area. The inclusion of greater diversity and a large number of sample respondents led the research to reduce its bias towards only

one type of outcome, creating a base for valid results. The other technique used was to restrict the amount of information shared with the respondents to make sure that the research was not biased by preconceived notions of the respondents. These steps helped to establish the validity of the results gained, proving the accuracy of the qualitative research. Further, the validity of the items was established using a panel of experts who reviewed the interview guide to see to it that any statement that did not go well with the subject of the study was removed (Chetty & Thakur, 2020).

Ethical considerations

Approval was obtained from the Head. Teachers and students of the school. The following information was contained in the approval: (1) The purpose of the study and the participants' rights were explained to them in their mother tongue; (2) Participation was strictly voluntary by signing the consent form, and no participant was coerced to sign the consent form or participate in the study; (3) Participants were fully informed of their right to withdraw from the study at any time without reason; (4) Participants were fully informed that they would participate in class for two weeks under the instruction of a teacher; (5) Participants were fully informed that they would take assessments at the beginning and the end as well as answer questionnaires after the completion of the instruction; (6) The sites were written to for permission; (7) The privacy and confidentiality of participants were protected using pseudonyms.

RESULTS AND DISCUSSION

In this section, the three research questions have been presented. Each research question contains a set of theorems and their proofs. Research question 1 (RQ1) was obtained by an interview, research question 2 (RQ2) by observation checklist, and research question 3 (RQ3) through focus group discussion. In all the analyses, only the dominant responses were extracted to form the themes from the 20 boys and 22 girls.

RQ1: Ostentation contracts

In these contracts, the researcher generated four themes for the sum of interior and two for the sum of exterior angles on students who employ ostentation didactical contracts.

THEOREM: The sum of the interior angles of a triangle is 180°

Theme 1—Method 1
Proof. First, draw a parallel line to the triangle (see Figure 3). The corresponding angles are equal, so $\angle b$ and $\angle b$' are the same. Also, since the alternate angles are equal, the angles of $\angle c$ and $\angle c$' are the same. So, if we add up $\angle a$, $\angle b$, and $\angle c$, we get a straight line.

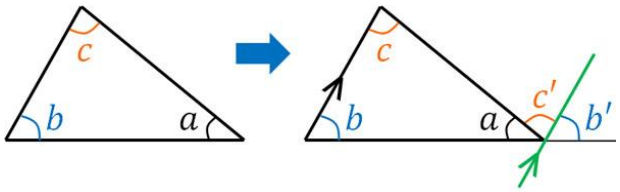

Figure 3. A line parallel to a side and through a vertex of the triangle

The angle of the straight line is 180°. Therefore, the sum of the interior angles of a triangle is 180°.

Theme 2—Method 2: The sum of the interior angles of a polygon is (n-2)180°
Proof. Any one point *P* inside the polygon, construct lines to the vertices (see Figure 4). There are altogether *n* triangles. Sum of angles of each triangle = 180°.

Please note that there is an angle at a point = 360° around *P* containing angles which are not interior angles of the given polygon. Sum of interior angles of *n*-sided polygon = $n\times180°- 360° = (n-2)\times180°$.

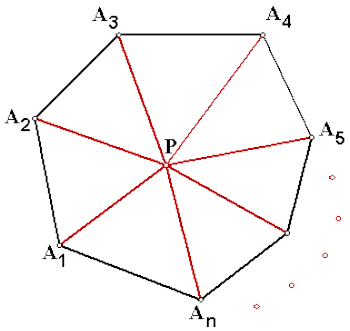

Figure 4. Point P inside the polygon

Theme 3—Method 3
Proof. The point *P* chosen may not be on the vertex, side, or inside the polygon (see Figure 5). It can even be a point outside the polygon.

There are altogether (*n*-1) triangles. Sum of angles of each triangle = 180°. Please note that the angles in triangle PA_1A_2 = 180° are not interior angles of the given

polygon. Sum of interior angles of *n*-sided polygon = (*n*-1)×180°- 180° = (*n*-2)×180°.

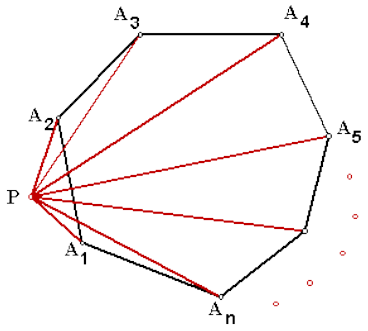

Figure 5. Point *P* was not on the vertex, side, or inside the polygon

Theme 4—Method 4

This method needs some knowledge of difference equations. It is a bit difficult but I think you are smart enough to master it.

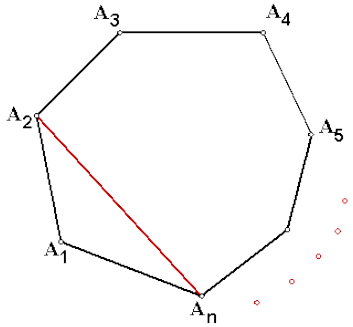

Figure 6. One vertex A_1 of a *n*-sided polygon

Proof. Let x_n be the sum of the interior angles of a *n*-sided polygon. So you may say that x_{n-1} is the sum of interior angles of an (*n*-1)—sided polygon. As in Figure 6, if you cut away one vertex, say A_1, of an *n*-sided polygon you can get an (*n*-1) sided polygon, $A_2A_3A_4\ldots A_n$. The angle sum of the triangle $A_1A_2A_n$= 180°. So, you get the difference equation: $x_n - x_{n-1}=180°$.

Similarly,

$$x_{n-1}-x_{n-2}=180°$$
$$x_{n-2}-x_{n-3}=180°$$
$$\ldots$$
$$x_4-x_3=180°$$

Lastly, we get the angle sum of a triangle $x_3=180°$. Adding up all the (*n*-2) equalities, and canceling all the terms, we get $x_n=(n-2)\times180°$.

THEOREM: The sum of exterior angles of a polygon is 360

Theme 1—Method 1

Proof. Before we carry on with our proof, let us mention that the sum of the exterior angles of an *n*-sided convex polygon = 360°. I would like to call this the Spider Theorem. Imagine (see Figure 7) you are a spider and you are now in point A_1 and facing A_2. You crawl to A_2 and turn an exterior angle, shown in red, and face A_3. You then crawl from A_2 to A_3 turn another exterior angle and face A_4. You carry on with the journey and turn all exterior angles.

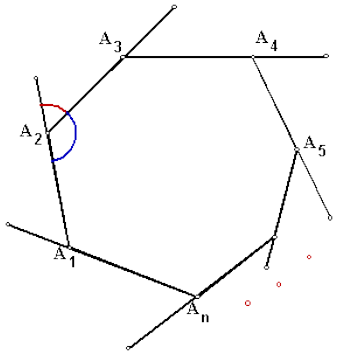

Figure 7. Spider crawl path

Lastly, you come back to point A_1 and face A_2 again. Aha! You have to turn a complete circle, which is 360°, and the intelligent spider has proved that the sum of the exterior angles of an *n*-sided convex polygon = 360°

Now, let us come back to our interior angles theorem. You can see that, by considering the red and blue angles in the diagram, the sum of any one of the interior angles and the adjacent exterior angle is 180° (adjacent angle on a straight line). There are *n*-sides in the polygon and therefore *n* straight angles. Sum of interior angles + sum of exterior angles = $n \times 180°$. Sum of interior angles + 360°= $n \times 180°$. Sum of interior angles = $n \times 180° - 360° = (n-2) \times 180°$.

Theme 2—Method 2

Proof. The sum of the exterior angles of a polygon having *n* sides. We know that, exterior angle + interior adjacent angle = 180°. So, if the polygon has n sides, then Sum of all exterior angles + Sum of all interior angles = $n \times 180°$. So, sum of all exterior angles = $n \times 180°$ - Sum of all interior angles. Sum of all exterior angles = $n \times 180° - (n-2) \times 180° = n \times 180° - n \times 180° + 2 \times 180° = 180°n - 180°n + 360° = 360°$. Therefore, we conclude that the sum of all exterior angles of the polygon having *n* sides = 360°.

RQ2: Mayeutic socrate contracts

In these contracts, the researcher generated two themes for the sum of interior and one for the sum of exterior angles on students' employment of Mayeutic Socratic contracts

THEOREM: The sum of the interior angles of any polygon is 180

Theme 1—Method 1

Proof. From any one of the vertices, say A_1, construct diagonals to other vertices (see Figure 8). There are altogether (n-2) triangles. Sum of angles of each triangle = 180°. Sum of interior angles of n-sided polygon = (n-2)×180°.

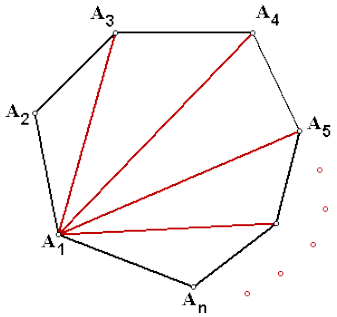

Figure 8. Vertex A_1 construct diagonals to other vertices

Theme 2—Method 2

Proof. From any point P on the line segment (see Figure 9), say A_1A_2, construct lines to the vertices A_3, A_4, …, A_n. There are altogether (n-1) triangles. Sum of angles of each triangle = 180°.

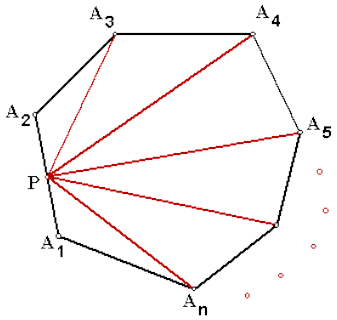

Figure 9. Point P on the line segment

Please note that there is a straight angle A_1PA_2 =180° containing angles that are not interior angles of the given polygon. Sum of interior angles of n-sided polygon = (n-1)×180°- 180° = (n-2)×180°.

THEOREM: The sum of the exterior angles of a polygon is 360

Theme 1—Method 1

Proof. Dotted lines of the same colour are parallel (see Figure 10). Clearly, from the right side of the diagram: $a + b + c + d + e = 360$

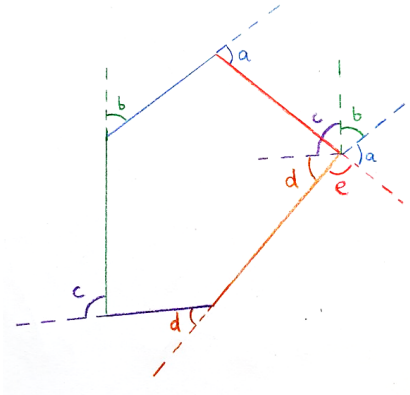

Figure 10. The extension of the sides of the polygon with dotted lines

Theme 2—Method 2

Proof. If a polygon is a convex polygon, then the sum of its exterior angles (one at each vertex) is equal to 360 degrees. Consider a polygon with n number of sides or an *n*-gon. The sum of its exterior angles is *N*. For any closed structure, formed by sides and vertex, the sum of the exterior angles is always equal to the sum of linear pairs and the sum of interior angles. Therefore, $N = 180n - 180(n\text{-}2) = 180n - 180n + 360 = 360$.

RQ3: Adidactical contracts

In these contracts, the researcher generated five adidactical contracts. The artifacts were analysed in the three stages of the didactical contract. The statement in each of the three stages satisfies the reasons the students use the adidactical contract.

In Figure 11, the participants in the focus group who comprised both boys and girls utilised three types of situations of action, formulation, and validation. The action situation helped them to determine the strategy, the formulation situation helped them to interact in class, and the validation situation equipped them to develop the arguments. In the cognitive hypotheses related to the materials, adaptation helped them to adjust themselves in the wake of contradictions, differences, and imbalances in the proofs. For instance, in the ostentation contracts, some participants believed that Theme 1 yielded better outcomes than Theme 2 because the triangle is similar to learning. Equally, in the Mayeutic Socrate contracts some participants accepted Theme 2 as compared to Theme

1. However, acculturation embedded in adidactical contracts helped the participants to resolve the differences and connect the construction of their knowledge with other participants in the group (Maknun et al., 2020).

Contract	Action	Formulation	Validation
	Conceptualizing cultural Adinkra objects that have triangular shapes	Drawing, tracing, counting sides, and measuring angles	Adding sides and angles to make 180° or 360°
	Abstracting Egyptian pyramid objects that have triangular shapes	Drawing, tracing, counting sides, and measuring angles	Adding sides and angles to make 180° or 360°
	Archaeologizing religious objects that have triangular shapes	Drawing, tracing, counting sides, and measuring angles	Adding sides and angles to make 180° or 360°
	Metallizing truss bridge objects that have triangular shapes	Drawing, tracing, counting sides, and measuring angles	Adding sides and angles to make 180° or 360°
	Roofing building objects that have triangular shapes	Drawing, tracing, counting sides, and measuring angles	Adding sides and angles to make 180° or 360°

Figure 11. Situations of actions, formulations, and validation

CONCLUSION

The researcher generated themes from the didactical ostentation, mayeutic Socratic, and potential adidactical contracts. These contracts are relational and hierarchical. While ostentation was purely the work of the teacher while students remained vigilant and keenly observant, mayeutic Socratic demanded that both the teacher and the students perform the tasks simultaneously. However, the teacher serves as a guide and the student an active deeper learner.

At the final adidactical contract stage, the participants carried the entire responsibility with little support from the teacher. The participants now had resorted to any teaching and learning resource in their immediate environment that could help achieve the results in minimum time and space. It was clear that even though truss bridges and roofing styles were not indigenous originations of their environment they were paramount to achieving the set goals. However, the Adinkra, Larabanga mosque, and Egyptian pyramids became their nearest artifacts.

It was therefore recommended that among other things: (1) Teachers streamlined method-friendly guidelines, strategies, and techniques at the ostentation contract for students to emulate; (2) There must be total cooperation and support of both the teacher and the student at the mayeutic Socratic contract to achieve mutual understanding and applications; (3) The knowledge of local indigenous and exotic artifacts must go hand in hand to facilitate independent and individualized students' studies and achievement.

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