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EXPLORING EXPONENTIAL FUNCTIONS USING GEOGEBRA

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Abstract

Exponential functions, symbolized by the expression $f(x)=a^x$, are a fundamental mathematical concept widely applicable to real-world scenarios such as population growth, compound interest, and radioactive decay. This abstract underscores the significance of exponential functions and highlights GeoGebra's crucial role in interactive visualization and analysis as an intuitive mathematical software. This study utilizes GeoGebra's dynamic interface to explore and comprehend exponential functions, bridging the gap between theoretical concepts and practical applications. It includes a comparative analysis between classical methods and GeoGebra solutions for exponential functions. The software's ability to validate mathematical outcomes through visual confirmation is explored, emphasizing its role in not only enhancing understanding but also providing a reliable means of verification. GeoGebra and mathematical analysis illustrations consistently yield results through practical examples, demonstrating the software's effectiveness in fostering a deeper understanding of exponential growth. In addition, surveys was conducted and engaged in direct comparison of solutions with students in the classroom setting to observe firsthand how learners interact with the material and identify common mistakes made during problem-solving. The survey results informed the development and refinement of the approach, ensuring a comprehensive understanding of both the benefits and challenges associated with learning exponential functions through GeoGebra.

Keywords: Analysis, Exponential functions, GeoGebra, Visualisation.

Abstrak

Fungsi eksponensial, disimbolkan dengan ekspresi $f(x)=a^x$, adalah konsep matematika dasar yang luas penerapannya dalam skenario dunia nyata seperti pertumbuhan populasi, bunga majemuk, dan peluruhan radioaktif. Abstrak ini menekankan pentingnya fungsi eksponensial dan menyoroti peran penting GeoGebra dalam visualisasi interaktif dan analisis sebagai perangkat lunak matematika yang intuitif. Studi ini menggunakan GeoGebra untuk menjelajahi dan memahami fungsi eksponensial, menjembatani kesenjangan antara konsep teoretis dan aplikasi praktis. Ini mencakup analisis perbandingan antara metode klasik dan solusi GeoGebra untuk fungsi eksponensial. Kemampuan perangkat lunak untuk memvalidasi hasil matematika melalui konfirmasi visual dieksplorasi, menekankan perannya tidak hanya dalam meningkatkan pemahaman tetapi juga memberikan cara verifikasi yang reliabel. Ilustrasi GeoGebra dan analisis matematis memberikan hasil yang konsisten melalui contoh praktis, menunjukkan efektivitas perangkat lunak dalam mendorong pemahaman yang lebih mendalam tentang pertumbuhan eksponensial. Selain itu, survei dilakukan dan melibatkan perbandingan langsung solusi dengan siswa di kelas untuk mengamati langsung bagaimana peserta didik berinteraksi dengan materi dan mengidentifikasi kesalahan umum selama pemecahan masalah. Hasil survei menginformasikan pengembangan dan penyempurnaan pendekatan ini, memastikan pemahaman yang komprehensif tentang manfaat dan tantangan yang terkait dengan pembelajaran fungsi eksponensial melalui GeoGebra.

Kata kunci*: *Analisis, Fungsi eksponensial, GeoGebra, Visualisasi.

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INTRODUCTION

Exponential functions are used in modeling many real-world phenomena, such as the growth of a population, the growth of an investment that earns compound interest, or the decay of a radioactive substance. Once an exponential model has been obtained, we can use the model to predict the size of a population, calculate the amount of an investment, or find the amount of a radioactive substance that remains (Stewart et al., 2018; Stewart, 2015; Larson & Edwards, 2018).

Exponential functions, symbolized by the mathematical expression $f(x)=a^x$, represent a fundamental and versatile concept in the realm of mathematics with wide-ranging applications in diverse fields (Lumen Learning, 2008; Marecek & Anthony-Smith, 2023). These functions, characterized by their rapid growth or decay, play a crucial role in modeling dynamic processes. In this project, we embark on an illuminating exploration of exponential functions, shedding light on their essential properties, and introduce GeoGebra as an integral tool for enhanced comprehension and interactive visualization. The technology and the time we are living make this easier because we can also use the resources from the internet for additional lessons and to see various illustrations and videos that explain the meaning of fractions, operations with them in more understandable and attractive way for the students and help them a lot to obeying the rules and avoiding the mistakes (Kamberi et al., 2022). The visualization and illustrations have greater impact compared with using of the program GeoGebra and Mathematica for solving exercises by students then solving exercises on the whiteboard, since students of lower classes hardly find themselves when working with GeoGebra, compared to higher classes (Mollakuqe et al., 2021). Except illustrations important role in understanding plays tasks, classroom commitment, extra hours, and extra exercises (Aliu et al., 2021).

This project aims to provide a comprehensive understanding of exponential functions while demonstrating the pivotal role GeoGebra, a dynamic and accessible mathematical software, plays in facilitating the exploration of these functions. We will delve into the core properties of exponential functions, examine their graphical representations, and delve into their real-world applications.

Through the integration of GeoGebra, we will showcase how this user-friendly software empowers learners, educators, and researchers to interactively engage with exponential functions. GeoGebra's dynamic interface allows for the creation of custom graphs, manipulation of parameters, and the generation of interactive constructions that enhance the understanding of exponential phenomena (Hohenwarter & Preiner, 2017).

Whether you are a student seeking to grasp the essence of exponential growth, an educator looking to enhance your teaching toolkit, or a researcher in need of a versatile mathematical companion, GeoGebra offers an intuitive platform to explore and visualize exponential functions.

RESEARCH METHODS

This literature review employed a systematic approach to identify and analyze relevant scholarly works related to exponential functions. The following steps outline the research methods utilized for this comprehensive exploration: (1) Literature search, (2) Inclusion and exclusion criteria, (3) Data extraction and synthesis, (4) Quality assessment, and (5) Integration of GeoGebra exploration.

The first step was literature search. A thorough search was conducted across academic databases, including PubMed, IEEE Xplore, JSTOR, and Google Scholar. Keywords such as "exponential functions," "applications of exponential functions," and "GeoGebra in mathematics education" were employed to ensure a broad coverage of relevant literature.

The second step was inclusion and exclusion criteria. The initial search results were filtered based on predetermined inclusion and exclusion criteria. Peer-reviewed articles, books, and conference proceedings that provided in-depth insights into the theoretical foundation, graphical representation, practical applications, and computational tools associated with exponential functions were included.

The third step was data extraction and synthesis. Relevant information from the selected sources was extracted, including key theoretical concepts, methodologies employed by previous researchers, and practical applications discussed. The data were synthesized to create the foundation for this literature review.

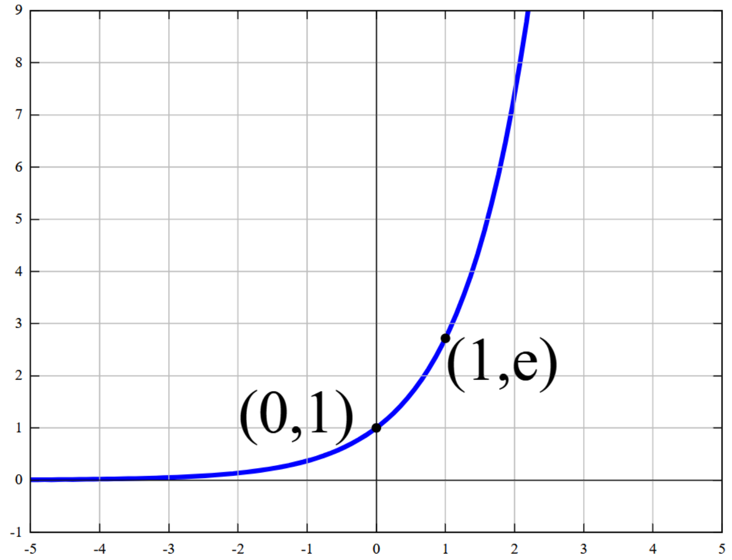
The forth step was quality assessment. The quality and reliability of the selected literature were assessed, taking into consideration factors such as the reputation of the authors, the publication venue, and the rigor of the methodologies employed in the original studies.

The fifth step was integration of GeoGebra exploration. Specific attention was given to literature discussing the use of GeoGebra in the context of exponential functions. Articles and studies highlighting the benefits and limitations of GeoGebra in enhancing the understanding and visualization of exponential phenomena were prioritized.

By employing these research methods within the literature review, this study aims to provide a comprehensive and well-informed exploration of the theoretical foundations, graphical representation, practical applications, and computational tools associated with exponential functions.

Exponential function

The exponential function is a mathematical function denoted by $f(x)=\exp(x)$ atau e^x, (where the argument x is written as an exponent) (BYJU'S, 2015). The term refers to the positive-valued function of a real variable, although it can be extended to the complex numbers or generalized to other mathematical objects like matrices or Lie algebras. The natural exponential function presented in Figure 1.



Source: https://en.wikipedia.org/wiki/Exponential_function

Figure 1. The natural exponential function along part of the real axis

The exponential function originated from the notion of exponentiation (repeated multiplication), but modern definitions (there are several equivalent characterizations) allow it to be rigorously extended to all real arguments, including irrational numbers. Its ubiquitous occurrence in pure and applied mathematics led mathematician Walter

(1987) to opine that the exponential function is the most important function in mathematics.

Exponential functions are expressed in the form $f(x)= a \cdot b^x$. In this expression: (1) $f(x)$ represents the value of the function at a given point x on the real number line; (2) a is a constant called the "initial value" or the "y-intercept" (the value of the function when x=0) (MATHguide, 2021); (3) b is a constant called the "base" of the exponential function, it is a positive real number greater than 0 and not equal to 1; (4) x is the independent variable, and it can take any real number.

There are several types of exponential functions based on the value of the base (MATHguide, 2017; BYJU'S, 2015; Marecek & Anthony-Smith, 2023). It is: (1) Growth exponential function (b>1), when the base b is greater than 1, the exponential function exhibits exponential growth, as x increases so $f(x)$ grows rapidly and without bound, for examples: 2^x, e^x, and 10^x, where e is Euler's number approximately equal to 2.71828; (2) Decay exponential function ($0<b<1$), when b is between 0 and 1, the exponential function exhibits exponential decay, as x increases so $f(x)$ approaches zero but never quite reaches it, for examples: $(1/2)^x$, 0.1^x, and $(1/e)^x$; (3) Identity function (b=1), when b is equal to 1, the exponential function simplifies to a constant function, in this case, $f(x)$= 1 for all values of x; (4) Negative base ($b<0$), exponential functions with negative bases are not commonly encountered in real-world applications because they can produce complex or imaginary values for certain exponents, for example: $(-2)^x$ can produce complex values for non-integer values of x; (5) Fractional exponents (b>0 and $b \neq 1$), exponential functions can also have fractional exponents, such as $b^{(1/2)}$ or $b^{(3/4)}$, these functions represent square roots, cube roots, and so on, depending on the value of the fractional exponent; (6) Zero base (b=0), when the base b is equal to 0, the exponential function is undefined for positive values of x, however 0^0 is typically defined as 1 in many mathematical contexts; (7) Exponential functions with negative exponents, exponential functions can have negative exponents, such as b^{-x}, these functions represent reciprocal values, where $f(x)$ gets smaller as x increases.

These are the main types of exponential functions based on the value of the base b. The behavior of the function, whether it grows, decays, remains constant, or behaves differently, is determined by the specific value of b in the function.

3rd year high school plan program for learning exponential functions

In this section, we will delve into the high school plan program in North Macedonia designed to teach students about exponential functions, their graphical representations, and solving exponential equations. This program is intended to help both teachers and students navigate the world of exponential functions effectively.

Program contents: (1) Defining exponential functions, students will start by understanding the fundamental concept of exponential functions and their basic structure; (2) Specifying the domain, emphasis will be placed on determining the valid domain of exponential functions to ensure a clear comprehension of their applicability; (3) Testing monotonicity, students will explore the monotonicity of exponential functions for different base values, enabling them to grasp the behavior of these functions; (4) Compiling function value tables, practicality is essential, students will actively compile tables of function values, gaining hands-on experience; (5) Constructing exponential function graphs, Using the data from function value tables, students will learn how to graph exponential functions, providing them with a visual representation of these functions' behavior; (6) Analyzing graphs, students will delve into the analysis of exponential function graphs using analytic notation to estimate the position and characteristics of the graph; (7) Defining exponential equations, introducing exponential equations, students will understand how to work with equations involving exponential functions; (8) Solving exponential equations, this section covers the solution of exponential equations, including linear, equations with functions, and quadratic exponential equations; (9) Discussion on exponential equation solutions, finally students will engage in discussions regarding the solutions of exponential equations, reinforcing their understanding.

Activity: Graphing exponential functions of the form

As part of the program, students will engage in hands-on activities involving the graphing of exponential functions with a base of 2, such as: $y=2^x$, $y=(1/2)^x$, $y=-2^x$, and $y=-(1/2)^x$ (Ministerstvo za Obrazovanie i Nauka, 2002; Nykamp, 2020; Marecek & Anthony-Smith, 2023). This exercise allows them to explore how changing the argument x, as well as the sign and value of the base, affects the shape of the exponential function.

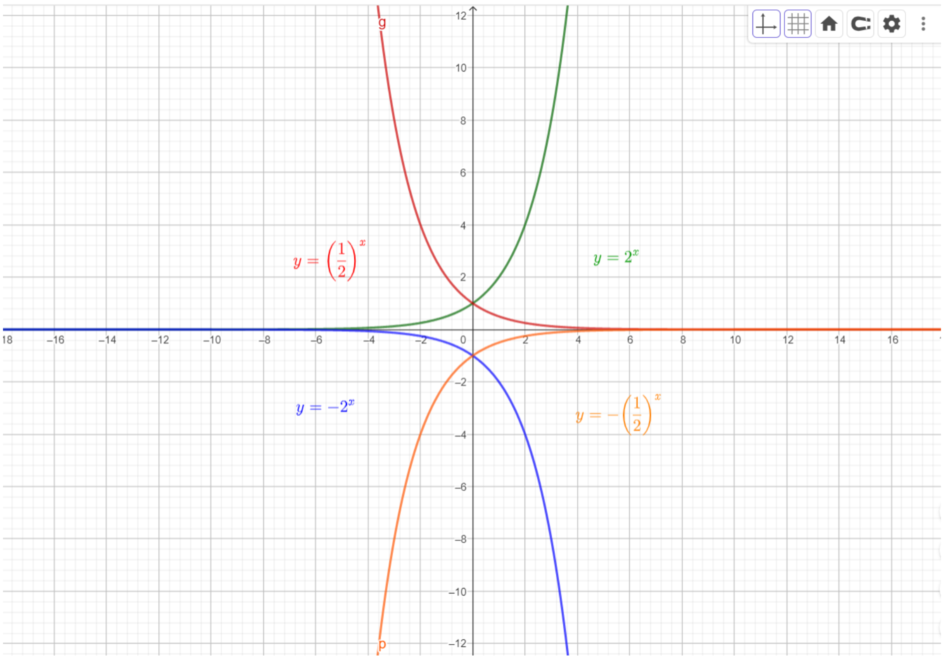
Exercises on solving exponential equations

To reinforce learning, students participate in exercises aimed at solving simple exponential equations. These exercises showcase the inverse operations of exponentiation and the necessity of introducing logarithmic operations.

Graphical representation of exponential functions

Exponential functions involve a base raised to a variable exponent. They exhibit unique behavior when graphed.

In Figure 2, it has a combined representation of four distinct exponential functions using GeoGebra, each color-coded for clarity. This composite graph allows us to visually compare and contrast different types of exponential behavior in a single view.



Source: https://www.geogebra.org/

Figure 2. Comparison of exponential functions: Growth, decay, and variations using GeoGebra.

The green color-coded graph, $y=2^x$, exhibits exponential growth. As x increases, the value of y grows rapidly. The graph is shown in green, illustrating the positive exponential trend. The red color-coded graph, $y=(1/2)^x$, represents exponential decay. As x increases, the value of y decreases significantly. This function models processes where values diminish over time. The bule color-coded graph, $y= -2^x$, shows exponential growth with a negative base. As x increases, y becomes increasingly negative. This function reflects scenarios involving negative growth or depreciation. The orange color-coded graph, $y = -(1/2)^x$, represents exponential decay with a negative sign. As x increases, y becomes more negative, demonstrating diminishing values over time. This function mirrors processes of negative exponential decay.

By examining these functions side by side, we gain valuable insights into how changes in base values and signs affect the shape and trend of exponential graphs. This visual comparison enhances our understanding of exponential functions and their real-world applications.

Real-world examples of exponential growth and decay

Exponential functions are commonly used to model population growth. As a population reproduces, the number of individuals increases exponentially until factors like limited resources or predation impose constraints. Additionally, in nuclear physics and radiology, exponential functions model the decay of radioactive substances over time (Nanda, 2023). It's crucial for determining the safety of nuclear materials and understanding decay rates.

Financial applications: Compound interest and investments

Exponential functions play a pivotal role in the world of finance (Keown, 2020). Compound interest formulas, often based on exponential growth, determine how investments grow over time (Turito, 2022; Nagwa, 2023). These formulas help individuals and financial institutions calculate future values of investments, loans, or savings accounts. Additionally, investments, investors use exponential growth models to assess the potential returns on investments (Chen, 2021; FlexBooks, 2022). Whether it's in stocks, bonds, or other financial instruments, understanding exponential growth is essential for making informed investment decisions.

Biological models: population growth and radioactive decay

In biology, exponential functions describe the growth of microorganisms, populations of species, or even the spread of diseases. Understanding growth rates is crucial for disease control, conservation efforts, and ecological studies. On the other hand, exponential decay models help biologists and medical researchers in applications such as radiometric dating and understanding the half-lives of drugs in the human body (Nagwa, 2023).

Engineering and physics applications: circuit analysis and half-life

Exponential functions find applications in electrical engineering, particularly in circuit analysis. They describe the charging and discharging of capacitors and inductors and help engineers design and troubleshoot electronic circuits. Moreover, in nuclear physics and chemistry, the concept of half-life is central (Nagwa, 2023). Exponential decay models help scientists predict the time it takes for a substance to reduce to half of its initial quantity, which has applications in fields like radiotherapy and carbon dating.

In these real-world scenarios, exponential functions are instrumental in understanding and predicting various phenomena, making them indispensable tools in fields ranging from biology and finance to physics and engineering.

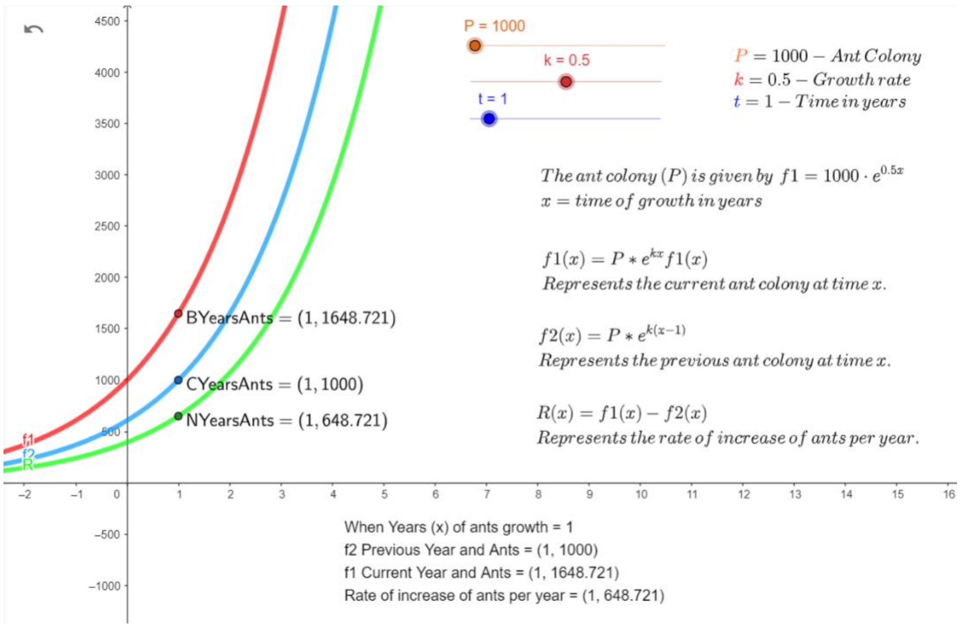
Using GeoGebra for plotting the exponential function

This function, $f(t)= a \ . \ e^{kt}$, represents an exponential growth or decay model, it's a mathematical model used to describe processes that exhibit exponential behavior, such as population growth, radioactive decay, or the charging or discharging of a capacitor in electronics. When it comes to solving exponential equations, two primary approaches emerge classical methods and software-based methods. Classical methods involve algebraic approaches and logarithmic techniques. These methods have been foundational in mathematics for centuries. They rely on manipulating equations symbolically to isolate the variable of interest. Key classical methods include algebraic approaches. Traditional algebraic techniques like factoring, exponent rules, and simplification are used to solve exponential equations.

Modern software and technology have revolutionized how we approach problem-solving. Software-based methods leverage computational tools to solve equations numerically and graphically. These methods offer advantages like speed, accuracy, and visualization. Notable software options include: (1) Mathematical software (like GeoGebra, Desmos, Mathematica, MATLAB, Maple etc., provide robust capabilities for solving complex mathematical equations, including exponential functions) and (2) Graphing calculators (advanced graphing calculators offer built-in functions for solving exponential equations, making them accessible for students and professionals).

Example

Calculate mathematically and present graphically (see Figure 3) in electronic form the growth of the colony of ants in their tropical habitat, calculating that in the first year t=1, the colony numbered P=1000 ants and the growth coefficient is k=0.5.



Source: https://www.geogebra.org/m/djxf7nuu

Figure 3. Exponential growth using GeoGebra

The formula used is $A(x)= P\ .\ e^{kx}$, where: (1) P represents the initial quantity or value at the starting point of the growth or decay process, it's the value at time $x = 0$; (2) e is the mathematical constant known as Euler's number, approximately equal to 2.71828, it is the base of the natural logarithm and is commonly used in exponential growth and decay equations; (3) k is the growth or decay rate constant, it determines the rate at which the quantity is changing, a positive k indicates exponential growth, and a negative k indicates exponential decay; (4) x is the independent variable, representing time or another relevant parameter that determines the point in time at which you want to calculate the quantity.

The formula is widely used in various fields. For example, in finance, it's used to model compound interest and investment growth. In population biology, it's used to model population growth or decline. In physics, it's used to describe processes like radioactive decay. The exact derivation may vary slightly depending on the specific application, but the fundamental concept of exponential growth or decay remains the same (Boyce & DiPrima, 2016).

RESULTS AND DISCUSSION

Together with 2nd-year high school students, we delved into the fascinating world of exponential functions, utilizing a range of teaching methods to enhance their understanding and engagement. The central focus of our session was to compare the effectiveness of the traditional classical method of teaching and the use of GeoGebra software. We presented mathematical problems related to exponential functions using both approaches, allowing these 2nd-year high school students to experience both the conventional and technological ways of learning. This collaborative endeavor not only enriched the students' understanding of exponential functions but also highlighted the practical implications of incorporating technology into mathematical exploration.

GeoGebra visualisation

In this GeoGebra project, a dynamic model has been created to simulate the growth of an ant colony over time. The key variables in this model are the ant colony (P), the rate of growth (k), and the time in years (t). By adjusting these sliders, it can observe how the ant colony's population changes over time.

Three sliders was defined: (1) P (ants colony), this slider allows you to set the initial population of the ant colony, and making an adjustment allows us to observe how the colony's growth is influenced by this parameter; (2) k (rate of growth), the k slider controls the rate at which the ant colony grows, it can make the colony grow faster or slower by changing this parameter; (3) t (time in years), the time slider represents the number of years over which we want to observe the ant colony's growth, and adjusting this slider allows us to see how the population changes over time.

To represent the growth of the ant colony, it is using mathematical functions: (1) $f_1(x)= P \cdot e^{(kx)}$, this function calculates the current population of the ant colony at a given time (x years), it's based on the initial population (P), the rate of growth (k), and the time (x); (2) $f_2(x)= P \cdot e^{(k(x-1))}$, this function calculates the previous population of the ant colony at (x-1) years, it helps to compare the growth over time; (3) $R(x)= f_1(x) – f_2(x)$, the rate of increase of the ant population per year ($R(x)$) is determined by the difference between the current population ($f_1(x)$) and the previous population ($f_2(x)$). When adjusting the P, k, and t sliders, it can be observed how these changes influence the ant colony's growth.

What you can learn from the model?

Increasing the initial population (*P*) results in a higher colony size at the start, which influences the growth rate over time. Adjusting the growth rate (*k*) changes the rate at which the colony grows. Higher values of *k* result in faster population growth. By changing the time (*t*) slider, you can see how the population evolves over a specific period. This is particularly useful for predicting future colony sizes.

This GeoGebra project provides a dynamic and interactive way to explore the growth of an ant colony based on the initial population, growth rate, and time. By adjusting these sliders, you can gain insights into how these parameters affect the colony's growth and observe the rate of population increase per year. This model can be a useful tool for understanding and predicting the growth of real ant colonies or other populations that exhibit exponential growth patterns. Click the link (https://www.geogebra.org/m/djxf7nuu) to start experimenting with the sliders and see how the colony's population changes over time.

Using the same formulas, we performed classical calculations in our notebook. We also used a calculator to derive the results. In our study of ant colony growth, we explored the dynamics of population change using both analytical methods and the interactive GeoGebra simulation. This comparison allowed us to verify the accuracy of our mathematical model and visualize the results.

By comparing our analytical solutions with the values obtained from the GeoGebra simulation for the given parameters, *P*, *k*, and *t*, we found that the results align closely. This not only validates the accuracy of our mathematical model but also demonstrates the power of visualizing complex dynamics using interactive tools like GeoGebra (Zulnaidi et al., 2020). This approach offers a clear and intuitive way to understand the growth patterns of ant colonies and similar systems.

Findings about the common mistakes that students make based on comparative study of classical and GeoGebra solutions: (1) Inaccurate recognition of *y*-values (lack of understanding of basic operations, order of operations), (2) Inadequate representation of a point in the coordinate system, (3) Incorrect representation of the axis intercept in the numerical axis, (4) Errors in identifying asymptotes, (5) Incorrect drawing of the coordinate system (incorrect division of the segment).

After pinpointing common mistakes made by students in handling exponential functions, our next crucial step was to engage in a constructive discussion on how to circumvent these errors. To enhance *y*-value recognition, (1) Emphasize the importance of mastering basic operations and the correct order of operations, and (2) Provide targeted practice exercises to reinforce these fundamental mathematical concepts. To improve point representation, introduce visualization techniques and practice exercises to refine skills in presenting points graphically. To correct axis intercept representation, (1) Clarify the proper methods for representing intercepts on the numerical axis, and (2) Provide examples and walkthroughs to ensure students grasp the correct procedures. To refine asymptote identification, (1) Offer additional exercises specifically focused on identifying asymptotes, and (2) Encourage the use of multiple approaches to confirm the accuracy of asymptote determinations. To ensure accurate coordinate system drawing, (1) Guide students in correctly dividing segments when drawing the coordinate system, and (2) Facilitate interactive sessions where students can apply these principles in real-time.

The result of a survey-based analysis

The following section provides a summary of the survey results, giving us a glimpse into the preferences and learning experiences of these 2nd-year high school students. The survey results indicate that: (a) Students understand the lesson better with the classical method (8 out of 23 students, 34.783%) and GeoGebra (15 out of 23 students, 65.217%); and (b) Students found it easiest to solve mathematical problems with exponential functions using the classical method (6 out of 23 students, 26.087%) and GeoGebra (17 out of 23 students. 73.913%).

Through surveys and interactive discussions with students, a unanimous consensus emerged regarding the substantial benefits of using GeoGebra, particularly in the realm of exponential functions. The majority of students expressed a positive and favorable stance, affirming that GeoGebra played a pivotal role in enhancing their understanding and problem-solving skills related to exponential functions.

Students highlighted the dynamic interface of GeoGebra as a crucial factor, enabling them to visualize and interact with exponential concepts in a way that transcended traditional learning methods. The software's capability to dynamically illustrate the impact of different parameters, create custom graphs, and provide real-time feedback garnered high praise from the students (Carreira et al., 2016).

Furthermore, the survey responses underscored that GeoGebra not only made complex mathematical concepts more accessible but also fostered a sense of engagement and curiosity among the students. Many acknowledged that the hands-on experience with GeoGebra significantly contributed to a deeper and more intuitive grasp of exponential growth, as opposed to conventional classroom approaches.

Overall, the unanimous agreement among students on the efficacy of GeoGebra for understanding exponential functions stands as a testament to the software's valuable contribution to the learning experience, providing an enriching and empowering tool for mathematical exploration.

CONCLUSION

In conclusion, exploring exponential functions using GeoGebra is a dynamic and insightful journey into the world of mathematics. Exponential functions, characterized by their rapid growth or decay, play a pivotal role in modeling various natural phenomena, financial trends, and scientific processes.

Through GeoGebra, we can visualize, analyze, and understand exponential growth and decay like never before. This software not only allows us to observe the behavior of exponential functions but also experiment with various parameters, transforming, and translating these functions to gain a more profound insight into their properties.

As we harness the capabilities of GeoGebra to model, graph, and manipulate exponential functions, we not only enhance our mathematical skills but also cultivate a deeper understanding of the fundamental principles that govern our real dynamic world.

GeoGebra is a valuable tool that opens doors to a world of exponential possibilities in mathematics and beyond.

Through a practical example, we came to the conclusion that the results are the same from a mathematical and visual point of view. Meanwhile GeoGebra helps us follow the exponential function in a dynamic way.

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